# Non-uniform Circular-Shaped Antenna Array Design and Synthesis - A Multi-Objective Approach

Saurav Ghosh[1], Subhrajit Roy[1], Sk. Minhazul Islam[1] , Shizheng Zhao[2], Ponnuthurai Nagaratnam Suganthan[2], and Swagatam Das[1]

[1] Dept. of Electronics and Telecommunication Engg., Jadavpur University, Kolkata 700 032, India
[2] Dept. of Electronics and Electrical Engg., Nanyang Technological Univrsity
saurav_online@yahoo.in, {roy.subhrajit20,skminha.isl}@gmail.com, ZH0047NG@e.ntu.edu.sg, epnsugan@ntu.edu.sg, swagatamdas19@yahoo.co.in

**Abstract.** Design of non-uniform circular antenna arrays is one of the important optimization problems in electromagnetic domain. While designing a non-uniform circular array the goal of the designer is to achieve minimum side lobe levels with maximum directivity. In contrast to the single-objective methods that attempt to minimize a weighted sum of the four objectives considered here, in this article we consider these as four distinct objectives that are to be optimized simultaneously in a multi-objective (MO) framework using one of the best known Multi-Objective Evolutionary Algorithms (MOEAs) called NSGA-II. This MO approach provides greater flexibility in design by producing a set of final solutions with different trade-offs among the four objective from which the designer can choose one as per requirements. To the best of our knowledge, other than the single objective approaches, no MOEA has been applied to design a non-uniform circular array before. Simulations have been conducted to show that the best compromise solution obtained by NSGA-II is far better than the best results achieved by the single objective approaches by using the differential evolution (DE) algorithm and the Particle Swarm Optimization (PSO) algorithm.

## 1 Introduction

Antenna arrays have an important role in detecting and processing signals arriving from different directions. Nowadays, antenna arrays [16] are preferred over single element antenna due to the latter's limitations in directivity and bandwidth. Design of antenna arrays overcomes such defects by associating each antenna elements in various electrical and geometrical configurations. The basic need for design of antenna array structure is to find out the positions of array elements that produce a radiation pattern in a whole that closely matches the desired pattern [1]. Recently synthesis of linear array elements separated in a non-linear fashion became immensely popular among the researchers working in electromagnetic domain.

B.K. Panigrahi et al. (Eds.): SEMCCO 2011, Part II, LNCS 7077, pp. 223–230, 2011.

However, the researchers have also started to show special interest in the design of antenna arrays of different shapes such as the circular one which now find immense applications in sonar, radar, mobile, and commercial satellite communication systems [3, 4].Till now, several single-objective meta-heuristic algorithms such as real-coded Genetic Algorithm (GA) [5], PSO [6], DE [7] have been applied to design non-uniform circular antenna arrays with minimum SLL, maximum directivity and null control. Design of circular antenna arrays requires optimization of four objectives. In the single objective approach, these four separate objectives are combined through a weighted linear sum into a single objective function. However, the weighted sum method is not appropriate and the solution obtained depends hugely on the relative values of the weights. In this article we have tried to solve the non-uniform circular antenna array problem with a multi-objective (MO) approach using NSGA-II [8]. Already some MO-based approaches [15, 17] have been made successfully in the field of antenna design. Unlike single-objective optimization techniques that return only a single best solution, the MOEAs [18] generate a set of non-dominated solutions (the Pareto optimal set). In order to validate the MO design method, we undertake a comparative study over one instantiation of the design problem comprising 8 element circular array. The best compromise solution obtained by NSGA-II is compared with two single-objective algorithms, namely DE [9] and PSO [10]. The comparison indicates that NSGA-II yields much better solutions as compared to DE and PSO demonstrating the effectiveness of multi-objective approach over single-objective approaches.

## 2 General Description of NSGA-II

NSGA-II [8] is non-domination based genetic algorithm for multi-objective optimization which incorporates elitism and no sharing parameter needs to be chosen *a priori*. The population is initialized as usual. Once the population is initialized the population is sorted based on non-domination into each front. The first front being completely non-dominant set in the current population and the second front being dominated by the individuals in the first front only and the front goes so on. Each individual in each front are assigned rank (fitness) values or based on front in which they belong to. Individuals in first front are given a fitness value of 1 and individuals in second are assigned fitness value as 2 and so on. In addition to fitness value a new parameter called *crowding distance* is calculated for each individual. The crowding distance is a measure of how close an individual is to its neighbours. Large average crowding distance will result in better diversity in the population. Parents are selected from the population by using binary tournament selection based on the rank and crowding distance. An individual is selected in the rank is lesser than the other or if crowding distance is greater than the other. The selected population generates off-springs from crossover and mutation operators. The population with the current population and current off-springs is sorted again based on non-domination and only the best *N* individuals are selected, where *N* is the population size. The selection is based on rank and the on crowding distance on the last front.

## 3 Multi-Objective Formulation of the Design Problem

The circular antenna array, being considered here, is non-uniform and planar, i.e. the elements are non-uniformly spaced on a circle of radius $r$ in the $x-y$ plane, as depicted in Figure 1. The elements comprising the circular antenna array are assumed to be isotropic sources. Now to calculate the array factor, we need to know the parameters of the array. The required parameters are the excitation current amplitude $\left(I_n\right)$, phase $\left(\beta_n\right)$, the angular position $\varphi_n$ of the $n$-th element, and the circular arc separation between any two adjacent elements ( $d_n$ -the distance between elements $n$ and $n-1$).

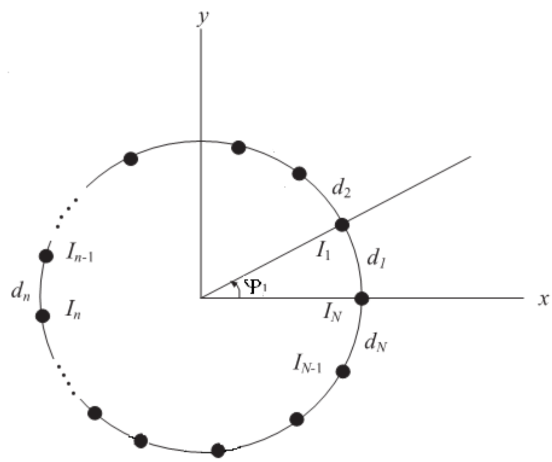


**Fig. 1.** Geometry of a non-uniform circular antenna array element with N isotropic radiators or elements

The expression for the array factor in the $x-y$ plane can be represented as:

$$AF\left(\varphi\right)=\sum_{n=1}^{N} I_n . e^{\,j.\left(kr.\cos\left(\varphi-\varphi_n\right)+\beta_n\right)} \tag{1}$$

Now $kr$ and $\varphi_n$ can be formulated as

$$\left.\begin{aligned} kr &= \frac{2\pi r}{\lambda} = \sum_{i=1}^{N} d_i, \\ \varphi_n &= \frac{2\pi}{kr}\sum_{i=1}^{n} d_i, \end{aligned}\right\} \tag{2}$$

In this article, our goal is to synthesize a circular antenna array with minimum side lobes level (SLL) and maximum directivity. The maximum side lobe level along with the average side lobe level is to be included in the fitness function for the directivity purposes. The following objective functions can be formulated in a mathematical form as shown below to satisfy these requirements:

$$f_{NU} = |AF(\varphi_{NULL1})| + |AF(\varphi_{NULL2})| \tag{3}$$

$$f_{SLA} = \frac{1}{\pi + \varphi_{NULL1}} \int_{-\pi}^{\varphi_{NULL1}} |AF(\varphi)| d\varphi + \frac{1}{\pi - \varphi_{NULL2}} \int_{\varphi_{NULL2}}^{\pi} |AF(\varphi)| d\varphi \tag{4}$$

$$f_{MSL} = |AF(\varphi_{MSLL1}| + |AF(\varphi_{MSLL2}| \tag{5}$$

where $\varphi_{NULL1}$ and $\varphi_{NULL2}$ are the two null angles, $\varphi_{MSLL1}$ is the angle where the maximum side lobe level (SLL) is obtained in the lower band $[-\pi, \varphi_{NULL1}]$ and, $\varphi_{MSLL2}$ is the angle where the maximum side lobe level (SLL) is obtained in the lower band $[\varphi_{NULL2}, \pi]$. In addition, to satisfy practical spatial requirements we incorporate another objective function which is the array circumference (since, planar array) that can be mathematically formulated as:

$$f_D = \sum_{i=1}^{N} d_i, \tag{6}$$

where, $d_i$'s is the distance between element $i$ and $i-1$, $i = 1,2,3,.....,N$. The equations 3, 4, 5 and 6 define the four objective functions to be optimized in a multi-objective approach, i.e. by a MOEA. While diminishing the array size one needs to keep in mind that the antenna directivity also decreases. An MOEA will allow us to find trade-off solutions between the four objectives shown above in order to achieve minimum side lobe level and maximum directivity with moderate circumference size. So an MOEA will provide greater flexibility in designing a non-uniform circular antenna array because a single-objective EA gives us only one solution in one run which might not completely satisfy the designer's needs.

## 4 Simulations and Results

One instantiation of the design problem, namely 8 element non-uniform circular antenna array is solved in a MO framework by using the NSGA-II algorithm. We compare the best compromise solution obtained by NSGA-II with the best results achieved by the single-objective optimization techniques, namely DE and PSO. This DE variant is known as DE/rand/1/bin and is the most popular one in DE literature [11]. The PSO variant used here is the original PSO algorithm as given in [10]. In case of single-objective optimization algorithms the objective function is the weighted sum of the four objectives considered here. In what follows we report the best results obtained from a set of 50 independent runs of NSGA-II and its single-objective competitors, where each run for each algorithm is continued up to 10000 Function Evaluations (FEs). Note that for NSGA-II we extract the best compromise solution obtained with the fuzzy membership function based method outlined in [13]. For NSGA-II and the contestant algorithms we employ the best suited parametric set-up chosen with guidelines from their respective literatures. To satisfy the requirements of practical considerations, current amplitudes are normalized with maximum value being set equal to 1.

Achieving the right balance between null control and side lobe level suppression has always been an issue in antenna design. So in Table 1 we provide the performance results of the two objectives $f_{NU}$ and $f_{SLA}$ for NSGA-II and the competitor single-objective algorithms DE and PSO over the single design instance which basically determines the balance between side lobe level suppression and null control. Table 2 presents the best values (out of 50 independent runs) of the two important factors which the designer is concerned of – Maximum SLL (in decibels), and Directivity (in decibels) obtained with the NSGA-II and the single-objective meta-heuristics DE and PSO. Figure 2 delineates the radiation patterns of the non-uniform circular antenna arrays generated by NSGA-II and all the other competitor algorithms for 8 element array.

The overall performance improvement regarding the circular antenna design can be derived from Table 2. A close scrutiny of Table 2 reveals that the NSGA-II yields much better values of two figures of merit – the maximum SLL (in dB) and the directivity (in dB) in comparison to the competitor algorithms. From Figure 2, it is clearly seen that the normalized array factor for the optimization parameter values achieved with NSGA-II has better side-lobe suppression than those of DE and PSO.

**Table 1.** Design objectives achieved with the three algorithms

| Number of Elements | Algorithm | $f_{NU}$ | $f_{SLA}$ |
|---|---|---|---|
| 8 | NSGA-II | **0.7360** | **0.2707** |
| | PSO | 0.8462 | 0.4315 |
| | DE | 0.8454 | 0.3820 |

**Table 2.** Design figures of merit obtained in the best (out of 50) run of the three algorithms on the 8 element array design instance

| Number of Elements | Algorithm | Maximum SLL (in dB) | Directivity (in dB) |
|---|---|---|---|
| 8 | NSGA-II | **-13.1** | **11.7276** |
| | PSO | -9.2 | 9.6936 |
| | DE | -10.7 | 10.2401 |

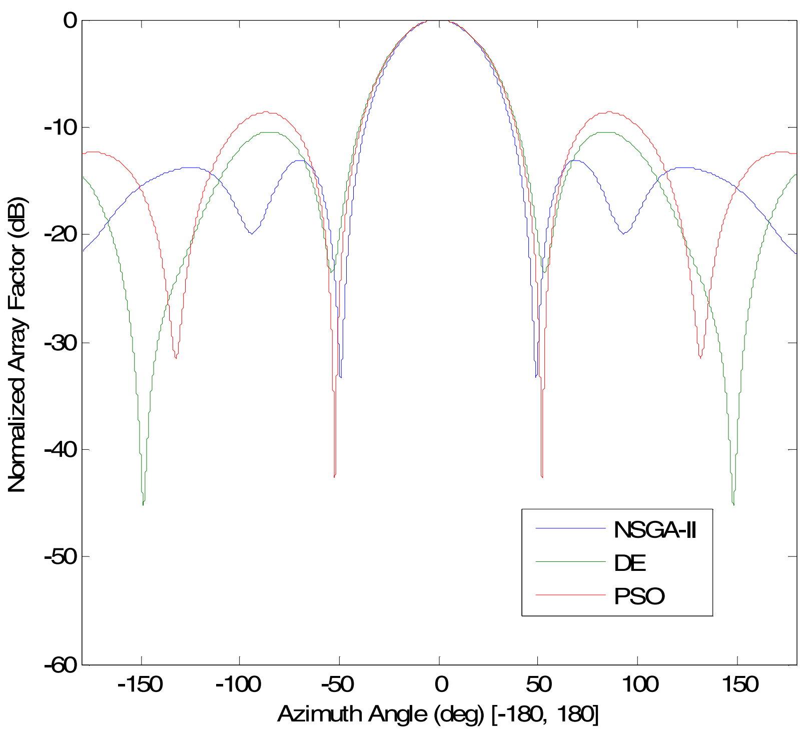


**Fig. 2.** Normalized Radiation Patterns obtained for 8 element circular antenna arrays using NSGA-II, DE and PSO

## 5 Conclusion

Synthesizing non-uniform circular antenna arrays with minimum SLL and maximum directivity has emerged as a popular as well as challenging optimization problem among the researches in electromagnetic domain. Contrary to the inefficient meta-heuristic single-objective approaches made before, in this article, we propose a multi-objective (MO) framework in which the four objectives associated with the circular arrays are optimized simultaneously with the help of NSGA-II algorithm. This MO framework provides immense advantages to the designer as the MOEAs generate the Pareto optimal set from which the designer can choose a desired solution and helps in finding the right balance between the four objectives. Our simulation experiments indicated that the best compromise solution obtained by NSGA-II algorithm could comfortably outperform the best results obtained with the traditional single-objective DE and PSO algorithms over the 8 element array design problem demonstrating the efficiency as well as effectiveness of the MO framework over the single objective one. NSGA-II is also able to achieve minimum SLL and maximum directivity among the competitors.

Future research may focus on designing other antenna array geometries as well as concentric arrays with the help of MO framework, in which different components of the cost function are treated as a multi-objective optimization problem. Other recent MO algorithms like 2LB-MOPSO [12], MODE-fast-sorting [14], MOEA/D-DE [2] as well as additional novel MO algorithms will be applied for solving these problems in order to comparatively evaluate different algorithms.

## References


1. Rahmat-Samii, Y., Michielssen, E. (eds.): Electromagnetic Optimization by Genetic Algorithms. Wiley, New York (1999)
2. Zhang, Q., Li, H.: MOEA/D: A Multiobjective Evolutionary Algorithm Based on Decomposition. IEEE Trans. Evolutionary Computation, 712–731 (2007)
3. Gurel, L., Ergul, O.: Design and simulation of circular arrays of trapezoidal-tooth log-periodic antennas via genetic optimization. Progress in Electromagnetics Research, PIER 85, 243–260 (2008)
4. Dessouky, M., Sharshar, H., Albagory, Y.: A novel tapered beamforming window for uniform concentric circular arrays. Journal of Electromagnetic Waves and Applications 20(14), 2077–2089 (2006)
5. Panduro, M., Mendez, A.L., Dominguez, R., Romero, G.: Design of non-uniform circular antenna arrays for side lobe reduction using the method of genetic algorithms. Int. J. Electron. Commun (AEU) 60, 713–717 (2006)
6. Shihab, M., Najjar, Y., Dib, N., Khodier, M.: Design of non -uniform circular antenna arrays using particle swarm optimization. Journal of Electrical Engineering 59(4), 216–220 (2008)
7. Panduro, M.A., Brizuela, C.A., Balderas, L.I., Acosta, D.A.: A comparison of genetic algorithms, particle swarm optimization and the differential evolution method for the design of scannable circular antenna arrays. Progress in Electromagnetics Research B 13, 171–186 (2009)

8. Deb, K., Pratap, A., Agarwal, S., Meyarivan, T.: A fast and elitist multiobjective genetic algorithm: NSGA-II. IEEE Transactions on Evolutionary Computation 6(2), 182–197 (2002)
9. Das, S., Suganthan, P.N.: Differential Evolution: A Survey of the State-of-the-Art. IEEE Transactions on Evolutionary Computation 15(1), 4–31 (2011), doi:10.1109/TEVC.2010.2059031
10. Kennedy, J., Eberhart, R.: Particle swarm optimization. In: Proceedings of the IEEE International Conference Neural Networks, vol. 4, pp. 1942–1948 (1995)
11. Price, K., Storn, R., Lampinen, J.: Differential Evolution - A Practical Approach to Global Optimization. Springer, Berlin (2005)
12. Zhao, S.Z., Suganthan, P.N.: Two-lbests Based Multi-objective Particle Swarm Optimizer. Engineering Optimization 43(1), 1–17 (2011), doi:10.1080/03052151003686716
13. Tapia, C.G., Murtagh, B.A.: Interactive fuzzy programming with preference criteria in multiobjective decision making. Comput. Oper. Res. 18, 307–316 (1991)
14. Qu, B.Y., Suganthan, P.N.: Multi-Objective Evolutionary Algorithms based on the Summation of Normalized Objectives and Diversified Selection. Information Sciences 180(17), 3170–3181 (2010)
15. Pal, S., Qu, B.Y., Das, S., Suganthan, P.N.: Optimal Synthesis of Linear Antenna Arrays with Multi-objective Differential Evolution. Progress in Electromagnetics Research, PIER B 21, 87–111 (2010)
16. Chandran, S. (ed.): Adaptive Antenna Arrays: Trends and Applications. Springer, Heidelberg (2004)
17. Pal, S., Das, S., Basak, A., Suganthan, P.N.: Synthesis of difference patterns for monopulse antennas with optimal combination of array-size and number of subarrays - A multi-objective optimization approach. Progress in Electromagnetics Research, PIER B 21, 257–280 (2010)
18. Zhou, A., Qu, B.-Y., Li, H., Zhao, S.-Z., Suganthan, P.N., Zhang, Q.: Multi-objective Evolutionary Algorithms: A Survey of the State-of-the-art. Swarm and Evolutionary Computation 1(1), 32–49 (2011)