Design of Two Channel Quadrature Mirror Filter Bank: A Multi-Objective Approach

Subhrajit Roy[1], Sk. Minhazul Islam[1], Saurav Ghosh[1], Shizheng Zhao[2], Ponnuthurai Nagaratnam Suganthan[2], and Swagatam Das[1]

[1] Dept. of Electronics and Telecommunication Engg., Jadavpur University, Kolkata 700 032, India
[2] Dept. of Electronics and Electrical Engg., Nanyang Technological Univrsity
{roy.subhrajit20,skminha.isl}@gmail.com, saurav_online@yahoo.in, ZH0047NG@e.ntu.edu.sg, epnsugan@ntu.edu.sg, swagatamdas19@yahoo.co.in

Abstract. In Digital Signal processing domain the Quadrature Mirror Filter (QMF) design problem is one of the most important problems of current interest. While designing a Quadrature Mirror Filter the goal of the designer is to achieve minimum values of Mean Square Error in Pass Band (MSEP), Mean Square Error in Stop Band (MSES), Square error of the overall transfer function of the QMF bank at the quadrature frequency and Measure of Ripple (*mor*). In contrast to the existing optimization-based methods that attempt to minimize a weighted sum of the four objectives considered here, in this article we consider these as four distinct objectives that are to be optimized simultaneously in a multi-objective framework. To the best of our knowledge, this is the first time to apply MO approaches to solve this problem. We use one of the best known Multi-Objective Evolutionary Algorithms (MOEAs) of current interest called NSGA-II as the optimizer. The multiobjective optimization (MO) approach provides greater flexibility in design by producing a set of equivalent final solutions from which the designer can choose any solution as per requirements. Extensive simulations reported shows that results of NSGA-II is superior to that obtained by two state-of-the-art single objective optimization algorithms namely DE and PSO.

1 Introduction

The basic idea and the layout of QMF bank was first proposed by Johnston [1]. Since then research concerning a QMF bank has been carried out and it has found wide applications in various signal processing fields [2-5].

Recently researchers have proposed many different techniques for the accurate design of QMF banks [6-10]. Finding the accurate filter coefficients of the QMF banks is a complex problem and traditional analytical methods may fail to solve this problem. Thus, the use of derivate free evolutionary algorithms (EA) seems to be a powerful alternative for the traditional methods to solve the QMF problem. The most

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popular EAs of current literature are Genetic Algorithms (GA) [11], Particle Swarm Optimization (PSO) [12], and Differential Evolution (DE) [13, 14]. These algorithms are suitable alternatives to the conventional methods because of their ability to deal with difficult problems featuring complex landscapes. These algorithms and their different variants find application in many real world problems [15, 16].

These single objective optimizers tackle all the objectives simultaneously by creating a single objective function by taking weighted sum of all of the objectives which may even be conflicting. In fact in [17] the QMF design problem has been tackled by the Particle Swarm Optimization technique where a weighted linear sum of all the design objectives was considered to form a single aggravated objective function. This weighted sum method is subjective and the solution obtained will depend on the values of the specified weights. It is difficult to find a universal set of weights that suits different instantiations of the same problem. So motivated by the inherent multi-objective nature of the QMF design problem and to remove the problems associated with using the weight factors we, to the best of our knowledge, for the first time use multi-objective algorithms [18] to solve this particular problem. Thus, we started to look for the most popular multi-objective algorithm that could solve this problem much more efficiently as compared to the conventional single-objective approaches. In this article we have employed the widely known NSGA-II algorithm [19] for the design of QMF banks.

In this work we employ NSGA-II to obtain the optimal coefficients of linear phase Quadrature Mirror Filter Bank while achieving the best possible design trade-offs between four objectives corresponding to minimum MSEP, minimum MSES, minimum Square error of the overall transfer function of the QMF bank at the quadrature frequency and minimum *mor*. The best tradeoff solutions, identified with a fuzzy membership based approach [20] over a certain instance of the design problem is shown to outperform the solutions achieved by single objective optimizers namely DE and PSO.

2 General Description of NSGA-II

NSGA-II [19] is a popular non-domination based genetic algorithm for multi-objective optimization which incorporates elitism and no sharing parameter needs to be chosen *a priori*. The population is initialized as usual. After the initialization of the population it is sorted based on non-domination into each front. The first front being completely non-dominant set in the current population and the second front being dominated by the individuals in the first front only and the front goes so on. Each individual of each of the front are assigned rank (fitness) values based on the front in which they belong to. Individuals in first front are assigned a fitness value of 1 and individuals in second are given fitness value as 2 and so on. In addition to fitness value a new parameter called *crowding distance* is calculated for each individual. The crowding distance is a measure of how close an individual is to its neighbours. Large average crowding distance will result in better diversity in the population. Parents are selected from the population by using binary tournament selection based on the rank and crowding distance. An individual is selected in the rank is lesser than the other or if crowding distance is greater than the other. The selected population generates off-springs from crossover and mutation operators. The population with the current population and current off-springs is sorted again based on non-domination and only the best *N* individuals are selected, where *N* is the population size. The selection is based on rank and the on crowding distance on the last front.

3 Multi-Objective Formulation of the Design Problem

The basic block diagram of a typical two-channel QMF bank is shown in the Figure1.

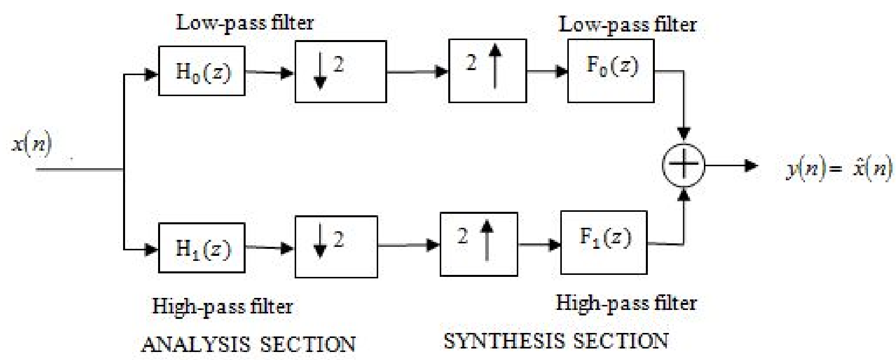

Fig. 1. Two Channel QMF Bank

This is basically a multi rate digital filter structure which splits the input signal $x(n)$ into two sub-band signals having equal band width, using the low-pass and high-pass analysis filters $H_o(z)$ and $H_1(z)$, respectively. These sub-band signals are down sampled by a factor of two to achieve signal compression or to reduce processing complexity. At the output side these two sub-band signals are interpolated by a factor of two and passed through low-pass and high-pass synthesis filters, $F_o(z)$and $F_1(z)$, respectively.

The z -transform of the output signal $x(n)$ of the two-channel QMF bank can be written as [6]:

$$\hat{X}(z)=\frac{1}{2}[H_0(z)F_0(z)+H_1(z)F_1(z)]X(z)+\frac{1}{2}[H_0(-z)F_0(z)+H_1(-z)F_1(z)]X(-z) \qquad (1)$$

The first term of the above equation represents a linear shift-invariant response between $X(z)$and $\hat{X}(z)$ whereas the second term represents the aliasing error because of change in sampling rate.

Thus the aliasing effect is removed if the second term of Equation 1 becomes zero i.e. if

$$[H_0(-z)F_0(z)+H_1(-z)F_1(z)]=0 \qquad (2)$$

This condition is simply satisfied by setting $F_0(z) = H_1(-z)$, $-F_0(z) = F_1(z)$ and $H_1(z) = H_0(-z)$. The Equation 1 reduces to

$$\hat{X}(z) = \frac{1}{2}[H_0(z)F_0(z) + H_1(z)F_1(z)]X(z) + \frac{1}{2}(0)X(-z) \tag{3.1}$$

or

$$\hat{X}(z) = \frac{1}{2}[H_0^2(z) - H_0^2(-z)]X(z) \tag{3.2}$$

or

$$\hat{X}(z) = T(z)X(z) \tag{3.3}$$

where,

$$T(z) = \frac{1}{2}[H_0^2(z) - H_0^2(-z)] \tag{4}$$

If we assume a linear-phase FIR low-pass filter with even length then the impulse response of the analysis section can be expressed as $h_0(n) = h_0(N-n-1)$ where $n = 0,1,2,\ldots(\frac{N}{2}-1)$, *N* is the length of the impulse response.

The frequency response of Equation 3 thus becomes

$$\hat{X}\left(e^{j\omega}\right) = \frac{1}{2}e^{-j(N-1)\omega}\left[\left|H_0(\omega)\right|^2 - (-1)^{N-1}\left|H_0(\pi-\omega)\right|^2\right]X\left(e^{j\omega}\right) \tag{5}$$

where

$$H_0(\omega) = \sum_{n=0}^{N/2-1} 2h_0(n)\cos(n - \frac{N-1}{2}) \tag{6}$$

The overall transfer function of QMF bank in frequency domain becomes

$$\frac{\hat{X}(e^{j\omega})}{X(e^{j\omega})} = T(e^{j\omega}) = \frac{1}{2}[|H_0(\omega)|^2 e^{-j\omega(N-1)} - |H_0(\omega-\pi)|^2 e^{-j(\omega-\pi)(N-1)}] \tag{7}$$

or,
$$T(e^{j\omega}) = \frac{1}{2}e^{-j\omega(N-1)}[|H_0(\omega)|^2 - (-1)^{N-1}|H_0(\omega-\pi)|^2] \tag{8}$$

or,
$$T(e^{j\omega}) = \frac{1}{2}e^{-j\omega(N-1)}T'(\omega) \tag{9}$$

where

$$T'(\omega) = [|H_0(\omega)|^2 - (-1)^{N-1}|H_0(\omega-\pi)|^2] \tag{10}$$

This reveals that the QMF bank has a linear phase delay due to the term $e^{-j\omega(N-1)}$ and the magnitude response $T'(\omega)$ should be unity for the condition of perfect reconstruction.

Then the condition for perfect reconstruction is to minimize the four objective functions E_p, E_s, E_t and *mor* defined below:

E_p is the mean square error in pass band (*MSEP*) which describes the energy of reconstruction error between 0 and ω_p.

$$E_p = \frac{1}{\pi} \int_0^{\omega_p} | H_0(0) - H_0(\omega) |^2 \, d\omega \tag{11}$$

E_s is the mean square error in stop band (*MSES*) which denotes the stop band energy related to LPF between ω_s and π [6] as ,

$$E_s = \frac{1}{\pi} \int_{\omega_s}^{\pi} (| H_0(\omega) |^2) d\omega \tag{12}$$

E_t is the square error of overall transfer function at quadrature frequency $\frac{\pi}{2}$.

$$E_t = \left[H_0\left(\frac{\pi}{2}\right) - \frac{1}{\sqrt{2}} H_0(0) \right]^2 \tag{13}$$

Measure of ripple (*mor*) [21] is

$$\text{mor} = \max | 10\log_{10} | T'(\omega) || - \min | 10 \log_{10} | T'(\omega) || \tag{14}$$

An MOEA will allow us to find the right balance between the four objectives shown above. When an MOEA is used then we get an approximation of the Pareto Front which contains numerous solutions. So an MOEA will allow us greater flexibility in designing a QMF bank because a single-objective EA gives us only one solution in one run which might not completely satisfy the designer's needs.

4 Experiments and Results

In this article, one instantiation of the design problem is solved in a MO framework by using the NSGA-II algorithm. We compare the best compromise solution obtained by NSGA-II with the best results achieved by the single-objective optimization techniques, namely DE and PSO. This DE variant is known as DE/rand/1/bin and is the most popular one in DE literature [13]. The PSO version used for comparison is the one used in the recently developed single objective QMF design problem employing PSO [17]. In what follows we report the best results obtained from a set of 50 independent runs of NSGA-II and its single-objective competitors, where each run for each algorithm is continued up to 10000 Function Evaluations (FEs). Note that for NSGA-II we extract the best compromise solution obtained with the fuzzy membership function based method outlined in [20]. For NSGA-II and the contestant algorithms we employ the best suited parametric set-up chosen with guidelines from their respective literatures.

We consider a two-channel QMF bank for $N = 24$, $\omega_p = 0.4\pi$, $\omega_s = 0.6\pi$ and with 12 filter coefficients. The performance of the considered algorithms are evaluated in terms of E_p, E_s, E_t, *mor,* stop-band edge attenuation (*SBEA*) and stop-band first lobe attenuation (*SBFLA*). E_p, E_s, E_t and *mor* are explained in Section 3, whereas SBEA= $-20\log_{10}(H_0(\omega_s))$ as given in [6] and *SBFLA* is obtained from the respective attenuation characteristics. One thing is to be noted that we want lower values of E_p, E_s, E_t and *mor* and higher values of *SBEA* and *SBFLA*.

In Table 1 we provide the values of objectives E_p, E_s, E_t, *mor* and other two performance indices *SBEA* and *SBFLA* obtained by NSGA-II, DE/rand/1/bin and PSO. Table 1 clearly depicts that the best compromise solution obtained by NSGA-II is better than that obtained by the single objective optimizers DE/rand/1/bin and PSO thereby proving the superiority of our MO approach over the single-objective approach. Figure 2a portrays the normalized amplitude response for H_o, H_1 filters of

Table 1. Design objectives, SBEA and SBLFA achieved with the three algorithms

Algorithms	E_p	E_s	E_t	*mor*	*SBEA (dB)*	*SBLFA (dB)*
NSGA-II	**1.72e-08**	**3.06e-06**	**9.18e-07**	**1.19e-02**	**23.061**	**34.983**
DE/rand/1/bin	2.18e-08	3.82e-06	5.43e-06	1.91e-02	19.664	32.853
PSO	2.35e-08	5.79e-06	7.65e-06	1.48e-02	22.783	34.431

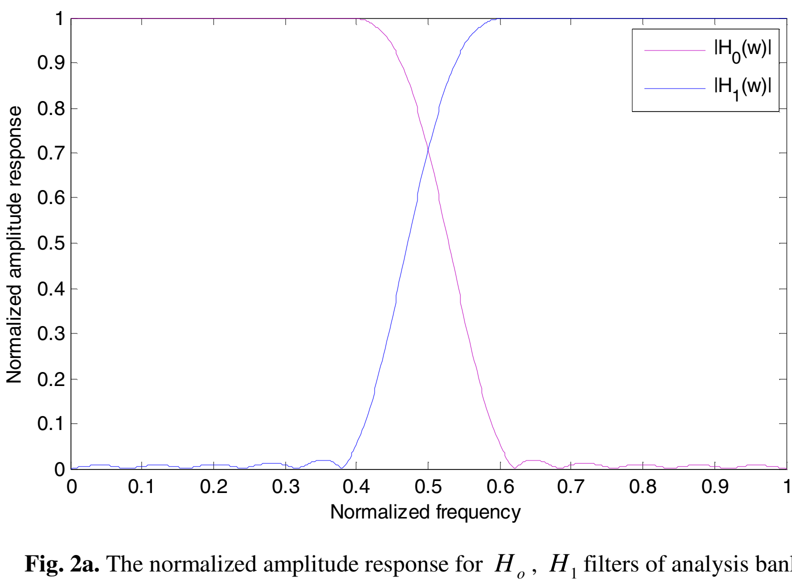

Fig. 2a. The normalized amplitude response for H_o, H_1 filters of analysis bank

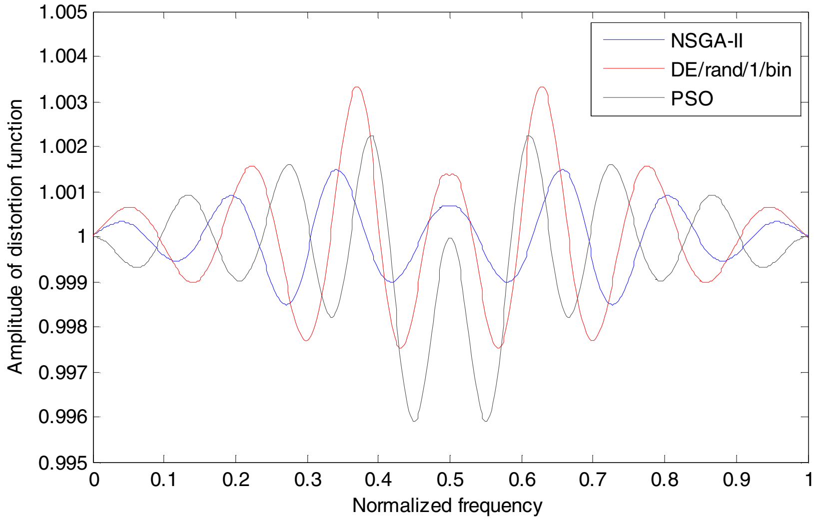

Fig. 2b. The attenuation characteristics of low pass filter H_o

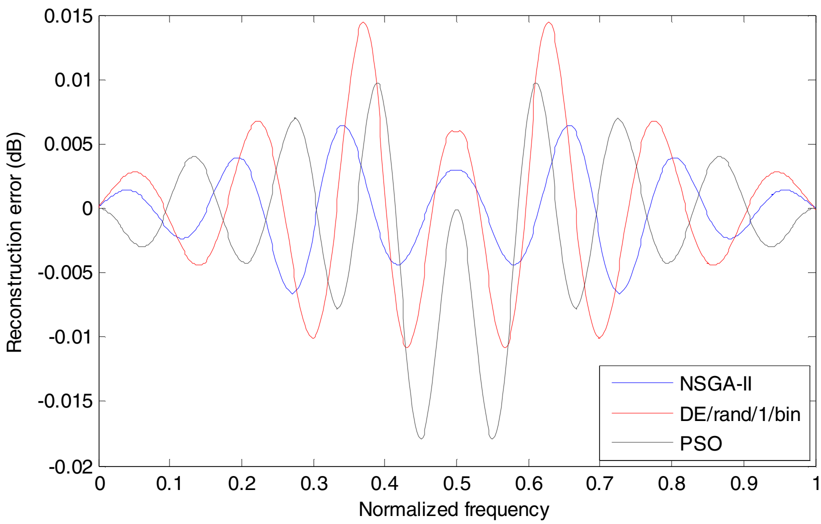

Fig. 2c. The reconstruction error of the QMF bank

analysis bank. The amplitude of distortion function $T'(\omega)$ for all the considered algorithms is shown in Figure 2b. It is evident from the figure that the amplitude distortion is lowest for the filter obtained by NSGA-II. The reconstruction error in dB of the QMF banks are plotted in Figure 2c which clearly shows that the reconstruction error is least for the filter obtained by NSGA-II algorithm.

5 Conclusion

In this article we have proposed a new technique for designing the QMF bank problem as a multi-objective optimization framework and have employed the NSGA-II algorithm to solve the problem. The formulated MO problem has four design objectives: Mean Square Error in Pass band (MSEP), Mean Square Error in Stop band (MSES), Square error of the overall transfer function of the QMF bank at the quadrature frequency and Measure of Ripple (*mor*). One instantiation of the QMF design problem has been considered as example in this article. The results obtained by NSGA-II have been compared with two state-of-the art single objective algorithms namely DE and PSO. The results obtained by the best compromise solution of NSGA-II have outperformed the results obtained by two state-of-the-art single objective optimization algorithms namely DE and PSO thereby proving the superiority of the MO framework of the QMF problem. Thus, unlike the single objective approaches the MO approach finally provide a set of design solutions that could allow the practitioner to satisfy the performance parameters. It gives a wide range of optimal solutions for the system under study. This MO framework is also robust and stable. This method is very effective and can be applied in practice to other filter design problems which are being treated till now as single objective problems. In our future work, we will investigate more design instances with other popular MO algorithms [22-24] as well as additional novel MO algorithms for solving this problem.

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